

Sample Paper – 4
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

- | | |
|--|--|
| <p>1. If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by x is less than y. The range of R is:</p> <p>(a) $\{1, 2, 3\}$ (b) $\{4, 6, 9\}$
(c) $\{13\}$ (d) None of these 1</p> | <p>7. If $(a + ib)^{1/3} = p + iq$, then $\frac{a}{p} + \frac{b}{q}$ is:</p> <p>(a) 0 (b) 1
(c) -1 (d) None of these 1</p> |
| <p>2. $(\sqrt{-4})(\sqrt{-2})$ is equal to:</p> <p>(a) $\sqrt{8}$ (b) $-\sqrt{8}$
(c) $i\sqrt{8}$ (d) None of these 1</p> | <p>8. Let $f(x) = \begin{cases} 3 - 2x, & x < 1 \\ 3x + 5, & 1 \leq x \leq 2 \\ 5x - 2, & x \geq 3 \end{cases}$
then $2f(0) + f(3) =$</p> <p>(a) 17 (b) 19
(c) 24 (d) 31 1</p> |
| <p>3. The largest coefficient in the expansion of $(a + b)^{18}$ is:</p> <p>(a) ${}^{18}C_{18}$ (b) ${}^{18}C_{12}$
(c) ${}^{18}C_9$ (d) ${}^{18}C_6$ 1</p> | <p>9. Let L, M, N be the feet of the perpendiculars drawn from a point $P(7.2, 9.3, 3.4)$ on the x, y and z-axes respectively. Find the coordinates of L, M and N respectively.</p> <p>(a) $(7.2, 0, 0), (0, 9.3, 0), (0, 0, 3.4)$
(b) $(7, 0, 0), (0, 0, 9), (0, 4, 0)$
(c) $(0, 7, 0), (0, 0, 9), (4, 0, 0)$
(d) $(0, 0, 7), (0, 9, 0), (4, 0, 0)$ 1</p> |
| <p>4. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is:</p> <p>(a) $\frac{3}{4}$ (b) $\frac{7}{5}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$ 1</p> | <p>10. Given the integers $r > 1, n > 2$, and coefficient of $(3r)^{\text{th}}$ and $(r + 2)^{\text{nd}}$ terms in the binomial expansion of $(1 + x)^{2n}$ are equal, then:</p> <p>(a) $n = 2r$ (b) $n = 3r$
(c) $n = 2r + 1$ (d) None of these 1</p> |
| <p>5. Let S be an infinite set and $S_1, S_2, S_3, \dots, S_n$ be sets such that $S_1 \cup S_2 \cup S_3 \cup \dots \cup S_n = S$ then</p> <p>(a) at least one of the sets S_i is a finite set
(b) not more than one of the set S_i can be infinite
(c) at least one of the sets S_i is an infinite set
(d) none of these 1</p> | <p>11. The symmetric difference of $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ is:</p> <p>(a) $\{1, 2\}$ (b) $\{1, 2, 4, 5\}$
(c) $\{4, 3\}$ (d) $\{2, 5, 1, 4, 3\}$ 1</p> |
| <p>6. The mean deviation from the mean of the set of observations $-1, 0$ and 4 is:</p> <p>(a) 3 (b) 1
(c) -2 (d) 2 1</p> | <p>12. If $-8 \leq 5x - 3 < 7$, then $x \in$:</p> <p>(a) $(-1, 2)$ (b) $[-1, 2)$
(c) $[-2, \infty)$ (c) $[-2, 0)$ 1</p> |

13. A pair of dice is rolled, if the outcome is such that the sum of numbers is 12, a coin is tossed. Then, the total number of outcomes for this experiment is:
 (a) 40 (b) 37
 (c) 41 (d) 43 1
14. The angle of a triangle are in A.P. and the ratio of angle in degree of the least to the angle in radians of the greatest is $60 : \pi$, find the angles in degrees.
 (a) $30^\circ, 60^\circ, 90^\circ$ (b) $40^\circ, 50^\circ, 90^\circ$
 (c) $30^\circ, 30^\circ, 120^\circ$ (d) $20^\circ, 130^\circ, 30^\circ$ 1
15. 18th term from the end of the sequence 3, 6, 12, ..., 25 terms is:
 (a) 393216 (b) 393206
 (c) 313216 (d) 303216 1
16. Consider the following data, the mean deviation about median for the data is:
- | | | | | | |
|-------|----|----|----|----|----|
| r_n | 15 | 21 | 27 | 30 | 35 |
| f_i | 3 | 5 | 6 | 7 | 8 |
- (a) 5 (b) 5.3
 (c) 5.1 (d) 5.2 1
17. If $S = \{x \mid x \text{ is a positive multiple of 3 less than } 100\}$ and $P = \{x \mid x \text{ is a prime number less than } 20\}$. Then, $n(S) + n(P)$ is equal to:
 (a) 34 (b) 31
 (c) 33 (d) 41 1
18. The number of six digit numbers whose all digits are odd is:
 (a) 6^5 (b) 5^6
 ..
 (c) $\frac{6!}{2!}$ (d) None of these 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.
19. Assertion (A): The foot of perpendicular drawn from the point $A(1, 2, 8)$ on the xy -plane is $(1, 2, 0)$.
 Reason (R): Equation of xy -plane is $y = 0$. 1
20. Assertion (A): A coin is tossed and then a die is rolled only in case a head is shown on the coin. The sample space for the experiment is
 $S = \{H1, H2, H3, H4, H5, H6, T\}$.
 Reason (R): 2 boys and 2 girls are in room X, and 1 boy and 3 girls are in room Y. Then, the sample space for the experiment in which a room is selected and then a person, is
 $S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$.
 where B_i denote the boys and G_j denote the girls. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. Find the value of $\tan 22^\circ 30'$.

OR

Convert following radian measure into degree measures $-\frac{2}{9}$. 2

22. Find $x^2 + 4ax + 4 > 0$ for all x . 2

23. Evaluate $\lim_{x \rightarrow 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ 2

24. Expand $\left(y^2 + \frac{2}{y}\right)^5, y \neq 0$.

OR

Which is larger $(1.01)^{1000000}$ or 10,000. 2

25. Find the sum to n terms of the G.P., whose k^{th} term is 5^k . 2



SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by $R = \{(x, y) : y = x - 1\}$. Write R in roster form. Write down the domain, co-domain and range of R . 3
27. A boy has 4 movie tickets and 9 movie of his interest in the theater. Of these 9, he does not want to see Marvels Part II, unless Marvels Part I is also seen. In how many ways can he choose the 4 movies to be seen?

OR

Evaluate:

(A) 6C_2

(B) ${}^{40}C_{40}$

(C) ${}^{13}C_3$

3

28. Evaluate: $\lim_{x \rightarrow 0} \left[\frac{\sin 5x - \sin 3x}{\tan 7x} \right]$. 3

29. Six new employees, two to whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?

OR

A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:

- (A) all the three balls are white.
 (B) all the three balls are red.
 (C) one ball is red and two balls are white. 3

30. If $\frac{2 \sin a}{1 + \cos a + \sin a} = y$, then prove that

$\frac{1 - \cos a + \sin a}{1 + \sin a}$ is also equal to y . 3

31. Find the derivative of $\frac{3x + 4}{5x^2 - 7x + 9}$.

OR

Find the derivation of $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$. 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. Prove that $\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 4\theta \sin \frac{7\theta}{2}$. 5

33. If $x + iy = \sqrt{\frac{p+iq}{a+ib}}$, prove that $(x^2 + y^2)^2 = \frac{p^2 + q^2}{a^2 + b^2}$. 5

34. Find the equation of the parabola whose vertex is the point (0, 2) and the directrix is the line $x + y - 3 = 0$.

OR

Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant. 5

35. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

Marks	0	1	2	3	4	5
Frequency	$x - 2$	x	x^2	$(x + 1)^2$	$2x$	$x + 1$

Where x is a positive integer. Determine the mean and standard deviation of the marks.

OR

Determine mean and standard deviation of first n terms of an A.P. whose first term is a and common difference is d . 5

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

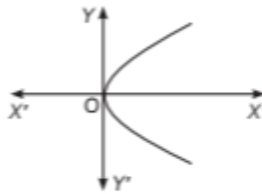
36. Case-Study 1:

A relation from a non-empty set A to a non-empty set B is said to be a function if every

element of set A has one and only one image in set B . In other words, we can say that a function f is a relation from a non-empty set A

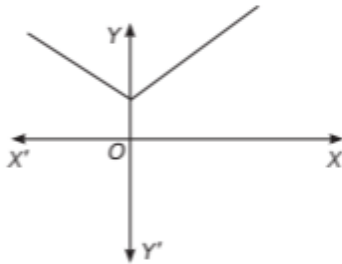
to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element or component. If f is a function from a set A to a set B , then we write $f: A \rightarrow B$ and it is read as f is a function from A to B or f maps A to B .

(A) Justify that the given curve represents a relation.



1

(B) Justify that the given curve represents a function.



1

(C) If $f(x) = x^2 + 2x + 3$, then among $f(1)$, $f(2)$ and $f(3)$, which one gives the maximum value.

OR

If $f(1+x) = x^2 + 1$, then find the value of $f(2-h)$.

2

37. Case-Study 2:

A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant. Rahul, being a plant lover, decides to open a nursery and he bought few plants and pots. He wants to place pots in such

a way that the number of pots in the first row is 2, in the second row is 4, in the third row is 8 and so on...

(A) What the constant multiple by which the number of pots is increasing in every row? 1

(B) Find the difference in number of pots placed in 7th row and 5th row. 1

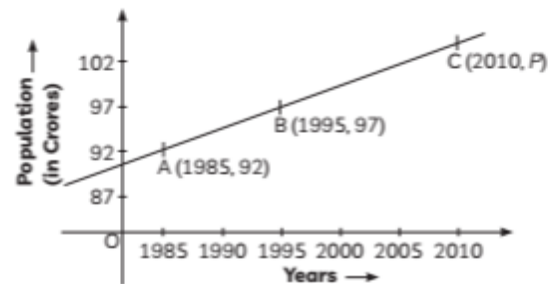
(C) Find the total number of pots upto 10th row.

OR

If Rahul wants to place 510 pots in total, then how many the total number of rows formed in this arrangement? 2

38. Case-Study 3:

India faces a major population crisis due to the growing population. If we were to estimate, we can say that almost 17% of the population of the world lives in India alone. India ranks second in the list of most populated countries. Population vs year graph given below:



Based on the above information, answer the

following questions.

(A) Find the population in the year 2010 (in crores). 2

(B) Find the equation of line perpendicular to line AB and passing through (1995, 97). 2

SOLUTION

SECTION - A

1. (b) {4, 6, 9}

Explanation: The relation from A to B is x is less than y .

Hence, Range is {4, 6, 9}

2. (b) $-\sqrt{8}$

Explanation: $(\sqrt{-4})(-\sqrt{2}) = i\sqrt{4} \cdot i\sqrt{2}$
 $= i^2 \sqrt{8} = -\sqrt{8}$

3. (c) ${}^{18}C_9$

Explanation: We know that if n is even then nC_r is greatest for $r = \frac{n}{2}$. Therefore ${}^{18}C_r$ is greatest for $r = 9$. Hence, the greatest coefficient is ${}^{18}C_9$.

4. (c) $\frac{2}{5}$

Explanation: Given that,

$$a = 3, b = -4, c_1 = 7, c_2 = 5$$

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|7 - 5|}{\sqrt{(3)^2 + (-4)^2}}$$

$$= \frac{2}{\sqrt{9 + 16}}$$

$$= \frac{2}{\sqrt{25}}$$

$$= \frac{2}{5}$$

5. (c) at least one of the sets S_i is an infinite set

Explanation: $S_1 \cup S_2 \cup S_3 \cup \dots \cup S_n$. For S to be an infinite set, at least one of sets S_i must be infinite, since if all S_i were finite, then S will also be finite.

6. (d) 2

Explanation: $\bar{x} = \frac{-1+0+4}{3} = 1$

$$\text{M.D.}(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{|-1-1| + |0-1| + |4-1|}{3} = 2$$

7. (d) None of these

Explanation: $a + ib = (p + iq)^3$
 $\Rightarrow a + ib = p^3 + (iq)^3 + 3p^2iq + 3p(iq)^2$
 $= p^3 - iq^3 + 3p^2qi - 3pq^2$
 $\Rightarrow a + ib = (p^3 - 3pq^2) + i(3p^2q - q^3)$
 Thus, $a = p^3 - 3pq^2, b = 3p^2q - q^3$

So,

$$\frac{a}{p} + \frac{b}{q} = \frac{p(p^2 - 3q^2)}{p} + \frac{q(3p^2 - q^2)}{q}$$

$$= p^2 - 3q^2 + 3p^2 - q^2$$

$$= 4(p^2 - q^2)$$

8. (b) 19

Explanation: Here,

$$f(0) = 3 - 2(0) = 3$$

$$f(3) = 5 \times 3 - 2 = 13$$

So,

$$2f(0) + f(3) = 2 \times 3 + 13$$

$$= 6 + 13$$

$$= 19$$

9. (a) (7.2, 0, 0), (0, 9.3, 0), (0, 0, 3.4)

Explanation: Since L is the foot of perpendicular from P on the x -axis, so its y and z -coordinates are zero. So, the coordinates of L is (7.2, 0, 0). Similarly, the coordinates of M and N are (0, 9.3, 0) and (0, 0, 3.4), respectively.

10. (a) $n = 2r$

Explanation: Given $(1 + x)^{2n}$

$$T_{3r} = T_{(3r-1)+1} = {}^{2n}C_{3r-1} x^{3r-1}$$

and $T_{r+2} = T_{(r+1)+1} = {}^{2n}C_{r+1} x^{r+1}$

Given, ${}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$$\Rightarrow 3r - 1 + r + 1 = 2n$$

$$[\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$$

Or, $n = 2r$

11. (b) {1, 2, 4, 5}

Explanation: $A \Delta B = (A - B) \cup (B - A)$

$$A - B = \{1, 2\}$$

$$B - A = \{4, 5\}$$

So, $(A - B) \cup (B - A) = \{1, 2, 4, 5\}$

12. (b) [-1, 2)

Explanation: We have, $-8 \leq 5x - 3 < 7$

$$\Rightarrow -5 \leq 5x < 10$$

$$\Rightarrow -1 \leq x < 2 \therefore x \in [-1, 2)$$

13. (b) 37

Explanation: The sample space associated with the given random experiment is

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6,H), (6,6,T)\}$

Hence, total number of sample points = 37

14. (a) $30^\circ, 60^\circ, 90^\circ$

Explanation: Let the angles of the triangle be $(a - d)^\circ, a^\circ, (a + d)^\circ$, where $d \geq 0$... (i)

Then $(a - d) + a + (a + d) = 180^\circ \Rightarrow a = 60$

\therefore From (i), the angles are $(60 - d)^\circ, 60^\circ, (60 + d)^\circ$

Now, the least angle = $(60 - d)^\circ$

And the greatest angle = $(60 + d)^\circ$

$$= (60 + d)^\circ \times \frac{\pi}{180} \text{ radian}$$

By the given condition, we have

$$\frac{60 - d}{\frac{\pi}{180}(60 + d)} = \frac{60}{\pi}$$

$$\Rightarrow \frac{180(60 - d)}{(60 + d)} = 60$$

$$\Rightarrow 180 - 3d = 60 + d$$

$$\Rightarrow 4d = 120 \Rightarrow d = 30$$

\(\therefore\) From (i), the angles are

$$(60 - 30)^\circ, 60^\circ, (60 + 30)^\circ \text{ i.e., } 30^\circ, 60^\circ, 90^\circ$$

15. (a) 393216

Explanation: Here $a = 3, r = 2$

$$m = 18 \text{ and } n = 25$$

We know that if a sequence has n term then m^{th} term from end is equal to $(n - m + 1)$ term from beginning

$$\begin{aligned} a_8 &= ar^{8-1} = 3(2)^{8-1} \\ &= 3(2)^7 \\ &= 3 \times 128 \\ &= 384 \end{aligned}$$

16. (c) 5.1

Explanation:

x_i	f_i	c.f	$ x_i - M $	$f_i x_i - M $
15	3	3	$ 15 - 30 = 15$	45
21	5	8	$ 21 - 30 = 9$	45
27	6	14	$ 27 - 30 = 3$	18
30	7	21	$ 30 - 30 = 0$	0
35	8	29	$ 35 - 30 = 5$	40
	29			148

$$N = 29 \text{ (odd)}$$

c.f. = cumulative frequency

$$\begin{aligned} \therefore \text{Median} &= \left(\frac{N+1}{2}\right)^{\text{th}} \text{ term} \\ &= \left(\frac{29+1}{2}\right)^{\text{th}} \text{ term} \\ &= \left(\frac{30}{2}\right)^{\text{th}} \text{ term} \\ &= 15^{\text{th}} \text{ term} \end{aligned}$$

As, 15th term lies in cf of 21, so Median = 30.

$$\begin{aligned} \text{M.D (M)} &= \frac{\sum f_i(x_i - M)}{\sum f_i} \\ &= \frac{148}{29} = 5.1 \end{aligned}$$

17. (d) 41

Explanation: $S = \{x: x \text{ is a positive multiple of 3 less than } 100\}$

$$\Rightarrow S = \{3, 6, 9, 12, \dots, 99\}$$

$$\Rightarrow n(S) = 33$$

$P = \{x: x \text{ is a prime number less than } 20\}$

$$\Rightarrow P = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$\Rightarrow n(P) = 8$$

$$\therefore n(S) + n(P) = 33 + 8 = 41$$

18. (b) 5^6

Explanation: Since, six-digit numbers whose all digits are odd are to be formed, suggests that repetition of digits is a must as available digit are 1, 3, 5, 7 and 9.

Hence, the required number is 5^6 .

19. (c) A is true but R is false.

Explanation: We know that in xy -plane, z -coordinate is 0. So, coordinate of foot of perpendicular drawn from point A(1, 2, 8) on xy -plane is (1, 2, 0).

Equation of xy -plane is $z = 0$

\(\therefore\) Reason is wrong.

20. (b) Both A and R are true but R is not the correct explanation of A.

Explanation: The sample space is

$$S = \{H^1, H^2, H^3, H^4, H^5, H^6, T\}$$

Where, H and T represents head and tail respectively of a coin.

When the room is selected, then

X	Y
B_1, B_2 G_1, G_2	B_3 G_3, G_4, G_5

There are four possibilities for selection of a person from room x, which are B_1, B_2, G_1, G_2 , similarly, there will be four possibilities for room Y.

So, the sample space is

$$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$$

SECTION - B

21. Let, $22^\circ 30' = \frac{\theta}{2}$

$\therefore \theta = 45^\circ$

$$\tan 22^\circ 30' = \tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

$$= \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \frac{\sin \theta}{1 + \cos \theta}$$

$$\therefore = \frac{\sin 45^\circ}{1 + \cos 45^\circ} = \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2} + 1}$$

$$= \frac{1 \times (\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \sqrt{2} - 1$$

Hence, $\tan 22^\circ 30' = \sqrt{2} - 1$

OR

$$\frac{-2}{9} \text{ radian} = \frac{-2}{9} \times \frac{180}{\pi}$$

$$= -\left(2 \times \frac{20}{\pi}\right)$$

$$= -\left(2 \times \frac{20 \times 7}{22}\right)$$

$$= -\left(\frac{20 \times 7}{11}\right)$$

$$= -\left(\frac{140}{11}\right) = -12.7272^\circ$$

22. Given, $x^2 + 4ax + 4 > 0$

$$\Rightarrow x^2 + 4ax + 4a^2 + 4 - 4a^2 > 0$$

$$\Rightarrow (x + 2a)^2 + (4 - 4a^2) > 0$$

$$\Rightarrow 4 - 4a^2 > 0$$

$$\Rightarrow 4 > 4a^2$$

$$\Rightarrow 1 > a^2$$

$$\Rightarrow |a| < 1 \Rightarrow -1 < a < 1$$

23. LHL at $x \rightarrow 0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h)$$

$$= \lim_{h \rightarrow 0} f(-h)$$

$$= \lim_{h \rightarrow 0} \frac{|-h|}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{h}{-h}$$

$$= \lim_{h \rightarrow 0} -1$$

$$= -1$$

RHL at $x \rightarrow 0$

$$D \lim_{h \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h)$$

$$= \lim_{h \rightarrow 0} f(h)$$

$$= \lim_{h \rightarrow 0} \frac{|h|}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h}$$

$$= \lim_{h \rightarrow 0} 1$$

$$= 1$$

Since Left Hand Limit (LHL) \neq Right Hand Limit (RHL)

$\therefore \lim_{h \rightarrow 0} f(x)$ doesn't exist.

24. By using binomial theorem, we have

$$\left(y^2 + \frac{2}{y}\right)^5$$

$$= {}^5C_0 (y^2)^5 + {}^5C_1 (y^2)^4 \left(\frac{2}{y}\right)^1 + {}^5C_2 (y^2)^3 \left(\frac{2}{y}\right)^2 + {}^5C_3 (y^2)^2 \left(\frac{2}{y}\right)^3 + {}^5C_4 (y^2)^1 \left(\frac{2}{y}\right)^4 + {}^5C_5 (y^2)^0 \left(\frac{2}{y}\right)^5$$

$$= y^{10} + 5y^8 \left(\frac{2}{y}\right) + 10y^6 \left(\frac{2}{y}\right)^2 + 10y^4 \left(\frac{2}{y}\right)^3$$

$$+ 5y^2 \left(\frac{2}{y}\right)^4 + \left(\frac{2}{y}\right)^5$$

$$= y^{10} + 10y^7 + 40y^4 + 80y + \frac{80}{y^2} + \frac{32}{y^5}$$

OR

$$(1.01)^{1000000} = (1 + 0.01)^{1000000}$$

$$= {}^{1000000}C_0 + {}^{1000000}C_1 (0.01)$$

+ other positive terms

$$= 1 + 1000000 \times 0.01 + \dots$$

$$= 10001 + \dots$$

$$\text{Hence, } (1.01)^{1000000} > 10,000.$$

25. Given that k^{th} term of a G.P. is 5^k .

$$\therefore a_k = 5^k \quad \text{---(1)}$$

Putting $k = 1$ in (1) we get

$$a_1 = 5 \quad \text{---(2)}$$

Putting $k = 2$ in (1), we get

$$a_2 = 25$$

Then, the first term is $a = a_1 = 5$ and the

common ratio is $r = \frac{a_2}{a_1} = \frac{25}{5} = 5$

Hence, the sum of n terms of the G.P. is given by

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{5(5^n - 1)}{5 - 1} = \frac{5}{4}(5^n - 1)$$

SECTION - C

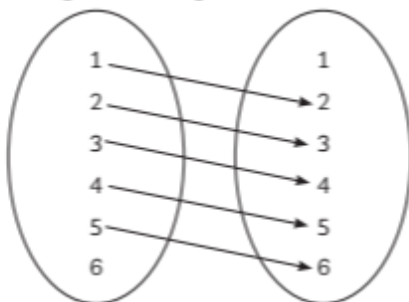
26. Given $A = \{1, 2, 3, 4, 5, 6\}$ and

$$R = \{(x, y) : y = x - 1\}.$$

The given relation in roster form can be written as

$$R = \{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$$

The given relation can be represented with the following arrow diagram:



So, domain of $R = \{1, 2, 3, 4, 5, 6\}$, co-domain of $R = \{0, 1, 2, 3, 4, 5\}$ and range of $R = \{2, 3, 4, 5, 6\}$.

27. To choose 4 movies from 9 movies. Marvel's Part II is not to be seen, unless Marvel's Part I is also seen.

Here, the order is not important.

So, each selection is a combination.

Case I: Marvels Part II is seen.

In this case, Marvels Part I is also seen.

Number of ways of selecting 2 movies from

$$7 \text{ movies} = {}^7C_2$$

$$= \frac{7!}{2!5!}$$

$$= 21$$

Case II: Marvels Part II is not seen.

In this case, Marvels Part I may or may not be seen.

Number of ways of selecting 4 movies from

$$8 \text{ movies} = {}^8C_4$$

$$= \frac{8!}{4!4!}$$

$$= 70$$

Hence, the required number of ways

$$= 21 + 70$$

$$= 91.$$

OR

$$\begin{aligned} \text{(A)} \quad {}^6C_2 &= \frac{6!}{2!(6-2)!} \\ &= \frac{6!}{2!4!} \\ &= \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad {}^{40}C_{40} &= \frac{40!}{40!(40-40)!} \\ &= \frac{40!}{40!0!} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(C)} \quad {}^{13}C_3 &= \frac{13!}{3!(13-3)!} \\ &= \frac{13!}{3!10!} \\ &= \frac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 1 \times 10!} \\ &= 286. \end{aligned}$$

$$28. \lim_{x \rightarrow 0} \left[\frac{\sin 5x - \sin 3x}{\tan 7x} \right] = \lim_{x \rightarrow 0} \left[\frac{\frac{\sin 5x}{x} - \frac{\sin 3x}{x}}{\frac{\tan 7x}{x}} \right]$$

[on dividing numerator and denominator by x]

$$\lim_{x \rightarrow 0} \left[\frac{5 \frac{\sin 5x}{5x} - 3 \frac{\sin 3x}{3x}}{7 \frac{\tan 7x}{7x}} \right]$$

$$= \frac{5 \lim_{x \rightarrow 0} \left[\frac{\sin 5x}{5x} \right] - 3 \lim_{x \rightarrow 0} \left[\frac{\sin 3x}{3x} \right]}{7 \lim_{x \rightarrow 0} \left[\frac{\tan 7x}{7x} \right]}$$

$$= \frac{5 \lim_{5x \rightarrow 0} \left[\frac{\sin 5x}{5x} \right] - 3 \lim_{3x \rightarrow 0} \left[\frac{\sin 3x}{3x} \right]}{7 \lim_{7x \rightarrow 0} \left[\frac{\tan 7x}{7x} \right]}$$

[$\because 5x \rightarrow 0, 3x \rightarrow 0, 7x \rightarrow 0$ as $x \rightarrow 0$]

$$= \frac{5(1) - 3(1)}{7(1)} = \frac{2}{7}$$

29. Let the couple occupied adjacent desks, consider those two as 1.

There are, $(4 + 1)$ i.e., 5 persons to be assigned.

\therefore Number of ways of assigning these five persons = $5! \times 2!$

Total number of ways of assigning 6 persons = $6!$

\therefore Probability that the couple has adjacent

$$\text{desks} = \frac{5! \times 2!}{6!} = \frac{1}{3}$$

Probability that the married couple will have

$$\text{non-adjacent desks} = 1 - \frac{1}{3} = \frac{2}{3}$$

OR

\therefore Number of red balls = 8

And number of white balls = 5

(A) P (all the three balls are white)

$$\begin{aligned} &= \frac{{}^5C_3}{{}^{13}C_3} = \left(\frac{\frac{5 \times 4}{2}}{\frac{13 \times 12 \times 11}{3 \times 2}} \right) \\ &= \frac{5 \times 4 \times 3}{13 \times 12 \times 11} = \frac{5}{143} \end{aligned}$$

(B) P (all the three balls are red)

$$\begin{aligned} &= \frac{{}^8C_3}{{}^{13}C_3} = \left(\frac{\frac{8 \times 7 \times 6}{3 \times 2 \times 1}}{\frac{13 \times 12 \times 11}{3 \times 2 \times 1}} \right) \\ &= \frac{8 \times 7 \times 6}{13 \times 12 \times 11} = \frac{28}{143} \end{aligned}$$

(C) P (one ball is red and two balls are white)

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{8 \times 10}{13 \times 12 \times 11} = \frac{40}{143}$$

30. Given, $\frac{2 \sin a}{1 + \cos a + \sin a} = y$

$$\text{Now, } \frac{1 - \cos a + \sin a}{1 + \sin a}$$

$$\begin{aligned} &= \frac{(1 - \cos a + \sin a)}{1 + \sin a} \cdot \frac{(1 + \cos a + \sin a)}{(1 + \cos a + \sin a)} \\ &= \frac{\{(1 + \sin a) - \cos a\} \cdot \{(1 + \sin a) + \cos a\}}{1 + \sin a} \\ &= \frac{(1 + \sin a)^2 - \cos^2 a}{(1 + \sin a)(1 + \cos a + \sin a)} \\ &= \frac{(1 + \sin^2 a + 2 \sin a) - \cos^2 a}{(1 + \sin a)(1 + \cos a + \sin a)} \end{aligned}$$

$$= \frac{(1 + \sin^2 a + 2 \sin a) - (1 - \sin^2 a)}{(1 + \sin a)(1 + \cos a + \sin a)}$$

$$= \frac{2 \sin^2 a + 2 \sin a}{(1 + \sin a)(1 + \cos a + \sin a)}$$

$$= \frac{2 \sin a (\sin a + 1)}{(1 + \sin a)(1 + \cos a + \sin a)}$$

$$= \frac{2 \sin a}{1 + \cos a + \sin a} = y$$

Hence, proved.

31. Let $y = \frac{3x + 4}{5x^2 - 7x + 9}$

Applying quotient rule of differentiation

$$\frac{d}{dx} \left(\frac{t}{s} \right) = \frac{s \cdot \frac{dt}{dx} - t \cdot \frac{ds}{dx}}{s^2}$$

Applying the rule

$$\begin{aligned} &(5x^2 - 7x + 9) \frac{d}{dx} (3x + 4) - (3x + 4) \\ \Rightarrow \frac{dy}{dx} &= \frac{\frac{d}{dx} (5x^2 - 7x + 9)}{(5x^2 - 7x + 9)^2} \\ \Rightarrow \frac{dy}{dx} &= \frac{3(5x^2 - 7x + 9) - (3x + 4)(10x - 7)}{(5x^2 - 7x + 9)^2} \\ &= \frac{(15x^2 - 21x + 27 - 30x^2 + 21x - 40x + 28)}{(5x^2 - 7x + 9)^2} \\ &= \frac{(-15x^2 - 40x + 55)}{(5x^2 - 7x + 9)^2} \\ &= \frac{(55 - 40x - 15x^2)}{(5x^2 - 7x + 9)^2} \end{aligned}$$

OR

$$\text{Let } y = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\Rightarrow y = \sqrt{\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}}$$

$$(\because 1 - \cos x = 2 \sin^2 \frac{x}{2}, 1 + \cos x = 2 \cos^2 \frac{x}{2})$$

$$\Rightarrow y = \sqrt{\tan^2 \frac{x}{2}}$$

$$\Rightarrow y = \tan \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \sec^2 \frac{x}{2} \cdot \left(\frac{1}{2} \right)$$

SECTION - D

32. L.H.S. = $\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2}$

$$= \frac{1}{2} \left[2 \cos \theta \cos \frac{\theta}{2} - 2 \cos 3\theta \cos \frac{9\theta}{2} \right]$$

$$= \frac{1}{2} \left[\cos \left(\theta + \frac{\theta}{2} \right) + \cos \left(\theta - \frac{\theta}{2} \right) - \left\{ \cos \left(3\theta + \frac{9\theta}{2} \right) + \cos \left(3\theta - \frac{9\theta}{2} \right) \right\} \right]$$

[$\because 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$]

$$= \frac{1}{2} \left(\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} - \cos \frac{15\theta}{2} - \cos \frac{3\theta}{2} \right)$$

$$= \frac{1}{2} \left(\cos \frac{\theta}{2} - \cos \frac{15\theta}{2} \right)$$

$$= -\frac{1}{2} \left[2 \sin \left(\frac{\theta+15\theta}{2} \right) \sin \left(\frac{2-152}{2} \right) \right]$$

[$\because \cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$]

$$= -\left\{ \sin 4\theta \sin \left(-\frac{7\theta}{2} \right) \right\}$$

$$= \sin 4\theta \cdot \sin \frac{7\theta}{2} \quad [\because \sin(-\theta) = -\sin \theta]$$

= R.H.S.
Hence, proved.

33. Given, $x + iy = \sqrt{\frac{p+iq}{a+ib}}$

$$\Rightarrow (x + iy)^2 = \left(\frac{p+iq}{a+ib} \right) \quad [\text{on squaring both sides}]$$

$$\Rightarrow |(x + iy)|^2 = \left| \frac{p+iq}{a+ib} \right|$$

[on taking modulus on both sides]

$$\Rightarrow |\sqrt{x^2 + (y)^2}|^2$$

$$= \frac{|p+iq|}{|a+ib|}$$

$$\Rightarrow (x^2 + y^2)^2 = \frac{p^2 + q^2}{a^2 + b^2} \quad [\text{on squaring both sides}]$$

⚠ Caution

$\hookrightarrow \left| \frac{a}{b} \right| = \frac{|a|}{|b|}$, where a, b are two non-zero complex numbers.
Hence proved.

34. Given, vertex of parabola is $(0, 2)$ and directrix of parabola is

$$x + y - 3 = 0 \quad \text{---(i)}$$

Then, slope of directrix = $-1 = m_1$ (say)

Since, axis of parabola is perpendicular to the directrix.

So, slope of axis = $-\frac{1}{m_1} = 1 = m_2$ (say)

Then, the equation of axis is of the form

$$y = m_2x + c, \text{ i.e., } y = x + c.$$

Since, axis passes through the vertex $(0, 2)$.

$$\therefore 2 = 0 + c$$

$$\Rightarrow c = 2$$

So, the equation of axis of parabola is

$$y = x + 2. \quad \text{---(ii)}$$

On solving (i) and (ii), we get $x = 1/2, y = 5/2$

The point of intersection of the directrix and axis is $D(1/2, 5/2)$.

Let, $F(\alpha, \beta)$ be the coordinates of the focus.

The mid-point of FD is $\left(\frac{\alpha+1/2}{2}, \frac{\beta+5/2}{2} \right)$

We know that the vertex V is the mid-point of FD .

$$\therefore \frac{\alpha+1/2}{2} = 0 \quad \text{and} \quad \frac{\beta+5/2}{2} = 2 \quad \text{i.e., } \alpha = -1/2$$

and $\beta = 3/2$.

So, the coordinates of focus of parabola are $(-1/2, 3/2)$.

Let, $P(x, y)$ be any point on the parabola.

Then, by definition of parabola, we have

Distance of $P(x, y)$ from $(-1/2, 3/2)$ = Perpendicular distance of $P(x, y)$ from $x + y - 3 = 0$

$$\Rightarrow \sqrt{(x+1/2)^2 + (y-3/2)^2} = \left| \frac{x+y-3}{\sqrt{(1)^2 + (1)^2}} \right|$$

$$\Rightarrow \sqrt{(x+1/2)^2 + (y-3/2)^2} = \left| \frac{x+y-3}{\sqrt{2}} \right|$$

$$\Rightarrow (x+1/2)^2 + (y-3/2)^2 = \frac{(x+y-3)^2}{2}$$

$$\Rightarrow 2(x^2 + x + 1/4 + y^2 - 3y + 9/4)$$

$$= x^2 + y^2 + 9 + 2xy - 6x - 6y$$

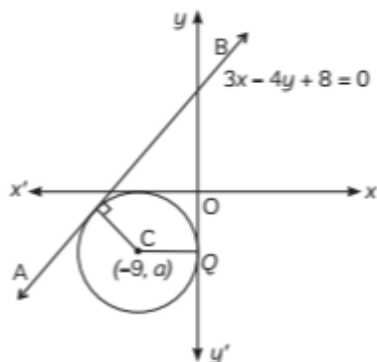
$$\Rightarrow x^2 - 2xy + y^2 + 8x - 4 = 0$$

Which is the required equation of parabola.

OR

Let a be the radius of the circle. Then, the coordinates of centre of the circle are $(-a, -a)$.

Now, perpendicular distance from C to the line AB = Radius of the circle.



$$d = \frac{|-3a + 4a + 8|}{\sqrt{9 + 16}} = \left| \frac{a + 8}{5} \right|$$

$$\Rightarrow a = \left| \frac{a + 8}{5} \right|$$

$$\Rightarrow a = \frac{a + 8}{5}$$

$$\Rightarrow 5a = a + 8$$

$$\Rightarrow 4a = 8$$

$$\Rightarrow a = 2$$

$$\text{Or } a = \frac{-(a + 8)}{5}$$

$$\Rightarrow 5a = -a - 8$$

$$\Rightarrow 6a = -8$$

$$\Rightarrow a = -8/6$$

But $a \neq -8/6$
 (\because Radius can't be negative)

So, the equation of circle is

$$(x + 2)^2 + (y + 2)^2 = 2^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 + 4y + 4 = 4$$

$$\Rightarrow x^2 + y^2 + 4x + 4y + 4 = 0$$

35. **Given:** There are 60 students in a class. The frequency distribution of the marks obtained by the students in a test is also given.

Now, we have to find the mean and standard deviation of the marks.

It is given there are 60 students in the class, so

$$\Sigma f_i = 60$$

$$\Rightarrow (x - 2) + x + x^2 + (z + 1)^2 + 2x + x + 1 = 60$$

$$\Rightarrow 5x - 1 + x^2 + x^2 + 2x + 1 = 60$$

$$\Rightarrow 2x^2 + 7x = 60$$

$$2x^2 + 7x - 60 = 0$$

Splitting the middle term, we get

$$\Rightarrow 2x^2 + 15x - 8x - 60 = 0$$

$$\Rightarrow x(2x + 15) - 4(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 4) = 0$$

$$\Rightarrow 2x + 15 = 0 \text{ or } x - 4 = 0$$

$$\Rightarrow 2x = -15 \text{ or } x = 4$$

Given x is a positive number, so x can take 4 as the only value.

And let assumed mean, $a = 3$

Now put $x = 4$ and $a = 3$ in the frequency distribution table and add other columns after calculation, we get

Marks (x_i)	Frequency (f_i)	$d_i = x_i - a$	$f_i d_i$	$f_i d_i^2$
0	$x - 2 = 2$	-3	-6	18
1	$x = 4$	-2	-8	16
2	$x^2 = 4^2 = 16$	-1	-16	16
3	$(x + 1)^2 = 25$	0	0	0
4	$2x = 8$	1	8	8
5	$x + 1 = 5$	2	10	20
Total	60		-12	78

And we know standard deviation is

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N} \right)^2}$$

Substituting values from the above table, we get

$$\sigma = \sqrt{\frac{78}{60} - \left(\frac{-12}{60} \right)^2}$$

$$\Rightarrow \sigma = \sqrt{1.3 - 0.04}$$

$$\Rightarrow \sigma = 1.26$$

Hence, the standard deviation is 1.26.

Now mean is

$$\begin{aligned} \bar{x} &= A + \frac{\Sigma f_i d_i}{N} = 3 + \left(\frac{-12}{60} \right) \\ &= 3 - \frac{1}{5} = \frac{14}{5} = 2.8 \end{aligned}$$

Hence, the mean and standard deviation of the marks are 2.8 and 1.26 respectively.

OR

Given first n terms of an A.P. whose first term is a and common difference is d .

Now, we have to find mean and standard deviation.

The given A.P. in tabular form is as shown below,

x_i	$d_i = x_i - a$	d_i^2
a	0	0
$a + d$	d	d^2
$a + 2d$	$2d$	$4d^2$
$a + 3d$	$3d$	$9d^2$
\vdots	\vdots	\vdots
$a + (n-1)d$	$(n-1)d$	$(n-1)^2 d^2$

Here we have assumed a as mean.

Given the A.P. have n terms, and we know the sum of all the terms of AP can be written as

$$\sum x_i = \frac{n}{2} [2a + (n-1)d]$$

Now we will calculate the actual mean,

$$\bar{x} = \frac{\sum x_i}{n}$$

Substituting the corresponding values, we get

$$\bar{x} = \frac{\frac{n}{2} [2a + (n-1)d]}{n}$$

The above equation can be written as

$$\bar{x} = \frac{[2a + (n-1)d]}{2}$$

$$\Rightarrow \bar{x} = a + \frac{[(n-1)d]}{2}$$

$$\Rightarrow \bar{x} = a + \frac{(n-1)d}{2}$$

We also have,

$$\sum d_i = \sum (x_i - a) = d[1 + 2 + 3 + \dots + (n-1)]$$

$$= d \left(\frac{n(n-1)}{2} \right)$$

$$\text{Also, } \sum d_i^2 = d^2 [1^2 + 2^2 + 3^2 + \dots + (n-1)^2]$$

$$= d^2 \left(\frac{n(n-1)(2n-1)}{6} \right)$$

Now we know standard deviation is given by

$$\sigma = \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n} \right)^2}$$

Substituting the corresponding values, we get

$$\sigma = \sqrt{\frac{d^2 \left(\frac{n(n-1)(2n-1)}{6} \right)}{n} - \left(\frac{d \left(\frac{n(n-1)}{2} \right)}{n} \right)^2}$$

$$\Rightarrow \sigma = \sqrt{d^2 \left(\frac{n(n-1)(2n-1)}{6n} \right) - d^2 \left(\frac{n^2 (n-1)^2}{4n^2} \right)}$$

$$\Rightarrow \sigma = \sqrt{d^2 \left(\frac{(n-1)(2n-1)}{6} \right) - d^2 \left(\frac{(n-1)^2}{4} \right)}$$

Taking out common terms we get

$$\sigma = \sqrt{\left(\frac{d^2 (n-1)}{2} \right) \left(\frac{(2n-1)}{3} - \frac{n-1}{2} \right)}$$

By taking the LCM, we get

$$\sigma = \sqrt{\left(\frac{d^2 (n-1)}{2} \right) \left(\frac{2(2n-1) - 3(n-1)}{6} \right)}$$

$$\Rightarrow \sigma = \sqrt{\left(\frac{d^2 (n-1)}{2} \right) \left(\frac{n+1}{6} \right)}$$

$$\Rightarrow \sigma = d \sqrt{\frac{(n^2 - 1)}{12}}$$

Hence, the mean and standard deviation of

the given A.P. is $a + \frac{(n-1)d}{2}$ and $d \sqrt{\frac{(n^2 - 1)}{12}}$ respectively.

SECTION - E

36. (A) If we draw a vertical line, then it will intersect the curve at two points. It shows that a given curve is a relation.

(B) If we draw a vertical line, then it will intersect the curve at only one point. It shows that a given curve is a function.

(C) $f(1) = 1 + 2 + 3 = 6$, $f(2) = 4 + 4 + 3 = 11$ and $f(3) = 9 + 6 + 3 = 18$. Here 18 is the maximum value. So, $f(3)$ gives the maximum value.

OR

$$\text{We have, } f(1+x) = x^2 + 1 \quad \text{---(i)}$$

On substituting $x = (1-h)$ in eq. (i), we get

$$f(1+1-h) = (1-h)^2 + 1$$

$$\Rightarrow f(2-h) = 1 + h^2 - 2h + 1 = h^2 - 2h + 2$$

37. (A) The number of pots in the successive rows are 2, 4, 8, ...

\therefore This forms a geometric progression,

$$\text{where } a = 2, r = \frac{4}{2} = 2$$

Hence, the constant multiple by which the number of pots is increasing in every row is 2.

(B) Number of pots in 7th row,

$a_7 = 2(2)^{7-1} = 2 \cdot 2^6 = 2^7 = 128$
 Number of pots in 5th row,
 $a_5 = 2(2)^{5-1} = 2 \cdot 2^4 = 2^5 = 32$
 Required answer = $128 - 32 = 96$
 (C) Total number of pots upto 10th row is

$$\begin{aligned}
 S_{10} &= \frac{a(r^{10} - 1)}{r - 1} = \frac{2(2^{10} - 1)}{2 - 1} \\
 &= \frac{2(1024 - 1)}{1} = 2046
 \end{aligned}$$

OR

Let there be n number of rows.

$$\therefore S_n = 510 = \frac{2(2^n - 1)}{2 - 1}$$

$$\begin{aligned}
 \Rightarrow \quad \frac{510}{2} &= 2^n - 1 \\
 \Rightarrow \quad 255 &= 2^n - 1 \\
 \Rightarrow \quad 256 &= 2^n \\
 \Rightarrow \quad 2^8 &= 2^n \\
 \Rightarrow \quad n &= 8
 \end{aligned}$$

 **Caution**

$$\rightarrow S_n = \frac{a(r^n - 1)}{r - 1} \text{ if } r > 1 \text{ only.}$$

38. (A) Let the population in year 2010 is P.

Since, A, B, C are collinear

\therefore Slope of AB = Slope of BC

$$\Rightarrow \frac{1}{2} = \frac{P - 97}{2010 - 1995}$$

$$\Rightarrow \frac{1}{2} = \frac{P - 97}{15}$$

$$\Rightarrow 7.5 = P - 97$$

$$\Rightarrow P = 97 + 7.5 = 104.5 \text{ crores}$$

(B) \therefore Slope of AB = $\frac{1}{2}$

Slope of line perpendicular to AB and

passing through (1995, 97) is

$$y - 97 = -2(x - 1995)$$

$$\Rightarrow y - 97 = -2x + 3990$$

$$\Rightarrow 2x + y = 4087$$

